SOFT PIONS AT HIGH ENERGY AND ITS PHENOMENOLOGICAL IMPLICATIONS

S.KORETUNE

Department of Physics, Shimane Medical University, Izumo, Shimane 693-8501, JAPAN
E-mail: koretune@shimane-med.ac.jp

The soft pion theorem in the inclusive reaction at high energy applied to the current induced reaction is explained briefly. A characteristic aspect of this theorem is the charge asymmetry produced by the pole terms in the soft pion limit. The pion charge asymmetry in the central region in the target-virtual-photon center of the mass (CM) frame of the semi-inclusive electroproduction and the contribution to the Gottfried sum are illustrated as examples.

1 Introduction

The soft pion theorem in the inclusive reaction at high energy was formulated by Sakai and Yamada[1] many years ago. Compared with the soft pion theorem in the exclusive reactions, the soft pion limit in the inclusive reaction can not be directly related to the physical processes without an additional assumption except in the neutral pion case. At present the fixed-mass sum rule approach[2] and the perturbative approach[3] are known. The former approach includes a new theoretical ingredient called as the current anticommutation relation on the null plane[4] and has been developed to the modified Gottfried sum rule[5] and its relatives.[6] The latter approach is an attempt to relate the structure function in the semi-inclusive reaction to the total inclusive reaction. By studying the semi-inclusive electroproduction, it was suggested that we can identify the soft pion at high energy as the directly produced pion with the low transverse momentum in the central region in the CM frame.[7] Further, it was shown that the theoretically expected value of the charge asymmetry in this kinematical region is very near to the experimental value.[3] Thus this phenomena may have some relevance to the Gottfried sum.[8] In this talk we explain these facts.

2 The current anticommutation relation on the null plane

The connected matrix element of the current anticommutation relation on the null plane in the flavor $SU(3) \times SU(3)$ model takes the following form.

$$\begin{aligned} &< p|\{J_a^+(x), J_b^+(0)\}|p >_c |_{x^+=0} \\ &=< p|\{J_a^{5+}(x), J_b^{5+}(0)\}|p >_c |_{x^+=0} \\ &= \frac{1}{\pi}P(\frac{1}{x^-})\delta^2(\vec{x}^{\perp})[d_{abc}A_c(p\cdot x, x^2=0) + f_{abc}S_c(p\cdot x, x^2=0)]p^+. \quad (1) \end{aligned}$$

Intuitively we can see that a physical origin of the factor $P(1/x^-)$ lies in the quantity $\partial\Delta^{(1)}(x)/\partial x^-$ at $x^+ = 0$. We can convince this fact very generally by using the Deser-Gilbert-Sudarshan(DGS) representation[9] for the current anticommutation relation which incorporates both the causality and the spectral condition.[4, 10] Then, based on this equation, the modified Gottfried sum rule is derived as[5]

$$\begin{aligned}
&\int_0^1 \frac{dx}{x}\{F_2^{ep}(x,Q^2) - F_2^{en}(x,Q^2)\} \\
&= \frac{1}{3}\left(1 - \frac{4f_K^2}{\pi}\int_{m_K m_N}^{\infty}\frac{d\nu}{\nu^2}\sqrt{\nu^2 - (m_K m_N)^2}\{\sigma^{K^+ n}(\nu) - \sigma^{K^+ p}(\nu)\}\right). \quad (2)
\end{aligned}$$

This sum rule explains the NMC deficit in the Gottfried sum[11] almost model independently. It has shown that the deficit is the reflection of the hadronic vacuum originating from the spontaneous chiral symmetry breaking. In this sense the physics underlining this algebraic approach has a common feature with that of the mesonic models reviewed in Ref.(12). However, in the algebraic approach, importance of the high energy region not only in the theoretical meaning but also in the numerical analysis has been made clear. In fact, it shows that about 40% of the NMC deficit comes from the region where the momentum of the kaon in the laboratory frame is above 4 GeV. Further the numerical prediction based on this sum rule exactly agrees with the recent experimental value from E866/NuSea collaboration.[13] On the other hand, a typical calculation in the mesonic model based on the πNN and the $\pi N\Delta$ processes accounts for about a half of the NMC deficit,[12] and also this model fairly well explains the experiment of E866/NuSea collaboration. These facts suggest that there may exist a dynamical mechanism to produce the flavor asymmetry at medium and high energy which we have overlooked as yet, and that it may compensate the above flaw of the mesonic models.

3 Soft pions at high energy

Usually, the soft pion theorem has been considered to be applicable only in the low energy regions. However in Ref.(1), it has been found that this theorem can be used in the inclusive reactions at high energy if the Feynman's scaling hypothesis holds. In the inclusive reaction "$\pi + p \to \pi_s(k) + anything$" with the π_s being the soft pion, it states that the differential cross-section in the CM frame defined as

$$f(k^3, \vec{k}^{\perp}, p^0) = k^0\frac{d\sigma}{d^3k}, \quad (3)$$

where p^0 is the CM frame energy, scales as

$$f \sim f^F(\frac{k^3}{p^0}, \vec{k}^{\perp}) + \frac{g(k^3, \vec{k}^{\perp})}{p^0}. \quad (4)$$

If $g(k^3, \vec{k}^{\perp})$ is not singular at $k^3 = 0$, we obtain

$$\lim_{p^0 \to \infty} f^F(\frac{k^3}{p^0}, \vec{k}^{\perp} = 0) = f^F(0,0) = \lim_{p^0 \to \infty} f(0,0,p^0). \tag{5}$$

This means that the π mesons with the momenta $k^3 < O(p^0)$ and $\vec{k}^{\perp} = 0$ in the CM frame can be interpreted as the soft pion. This fact holds even when the scaling violation effect exists, since we can replace the exact scaling by the approximate one in this discussion. The important point of this soft pion theorem is that the soft-pion limit can not be interchanged with the manipulation to obtain the discontinuity of the reaction "$a+b+\bar{\pi}_s \to a+b+\bar{\pi}_s$". We must first take the soft pion limit in the reaction "$a+b \to \pi_s + anything$". This is because the soft pion attached to the nucleon(anti-nucleon) in the final state is missed in the discontinuity of the soft pion limit of the reaction "$a+b+\bar{\pi}_s \to a+b+\bar{\pi}_s$". [1]

4 The charge asymmetry

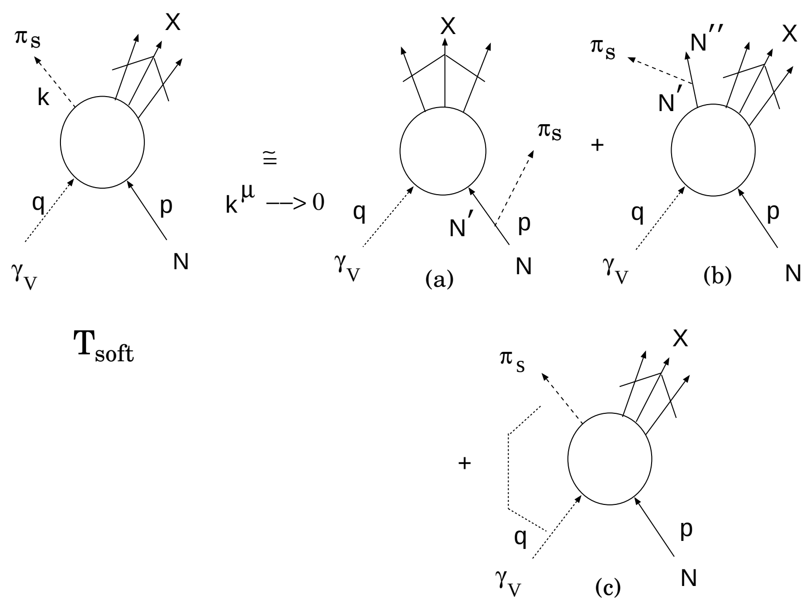

Figure 1: The amplitude in the soft pion limit. The graph (a) is the one coming from the soft pion emission from the initial nucleon, the graph (b) is the one from the final nucleon(anti-nucleon), and the graph (c) is the one from the commutation relation on the null plane.

The hadronic tensor is obtained by squaring the amplitude shown in Fig.1, and hence we have many cross terms. Further in some cases the pole term

is inhibited by the charge conservation. Because of this fact, the application of the soft pion theorem in the inclusive reaction to the charged pion case is non-trivial. In many cases the limit can not be related to the known process without an additional assumption. Here we use the light cone current algebra[14] at some Q_0^2 where the evolution is started. We must take care whether the quantity is singlet piece or non-singlet one and whether the quantity is charge conjugation even or odd. Then the perturbative information can be taken into account through the structure functions related by this way. The usefulness of this method is that we can use the symmetry relations at $Q^2 = Q_0^2$. Now the method in Ref.(1) had not been checked experimentally, it was done in the soft π^- case.[7] From the experimental data of the Harvard-Cornell group[15] the data satisfying the following condition are selected.

(1) The transverse momentum satisfies $|\vec{k}^{\perp}|^2 \leq m_\pi^2$.

(2) The change of F^- can be regarded to be small in the small x_F region.

The effective cut of x_F following the condition (2) is taken about at 0.2. Then the theoretical value is roughly estimated as 10% $\sim$ 20% of the experimental value. However, in the central region, there are many pions from the decay of the resonances, and about 20% $\sim$ 30% can be expected to be the pion from the directly produced pion. Hence the theoretical value is the same order with the experimental value. Now to reduce the ambiguity due to the pion from the resonance decay product, the charge asymmetry was calculated.[3] The theoretical value is roughly equal to $0.15 \sim 0.18$ with weak x_B dependence. While the experimental value[15] with the transverse momentum satisfying the condition (1) is almost constant in the region $0 < x_F < 0.1$ with its value 0.28 ± 0.05, and it gradually decreases above $x_F = 0.1$. The data depends on x_B weakly. Hence the theoretical value is very near to the experimental value.

5 The soft pion contribution to the Gottfried sum and its phenomenological implications

The soft pion contribution to the Gottfried sum has been estimated.[8] Adding the contributions from the soft π_s^+, π_s^-, and π_s^0, subtracting the contributions to F_2^{en} from those to F_2^{ep}, and using symmetric sea polarization for simplicity, we obtain

$$\begin{aligned}&(F_2^{ep} - F_2^{en})|_{soft}\\ &= \frac{I_\pi}{4f_\pi^2}[g_A^2(0)(F_2^{ep} - F_2^{en})(3 < n > -1) - 16xg_A(0)(g_1^{ep} - g_1^{en})], \qquad (6)\end{aligned}$$

where I_π is the phase space factor for the soft pion and $< n >$ is the sum of the nucleon and anti-nucleon multiplicity. To estimate the magnitude of this asymmetry, we approximate $F_2^{ep}, F_2^{en}, g_1^{ep}, g_1^{en}$ on the right-hand side of Eq.(6) by the valence quark distribution functions at $Q_0^2 = 4$ GeV^2.[16] As a

multiplicity of the nucleon and antinucleon, we set $< n >= a \log_e s + 1$, where $s = (p + q)^2$. The parameter a is fixed as 0.2 in consideration for the proton and the anti-proton multiplicity in the e^+e^- annihilation such that $\frac{1}{2} a \log_e s$ with $\sqrt{s}$ replaced by CM energy of that reaction agrees with it.[17] Following the experimental check of the charge asymmetry in the previous section, the transverse momentum is restricted by the condition (1) and the Feynman scaling variable is cut at $x_F = 0.1$. By calculating the phase space factor under these conditions and allowing a small change of the parameters to determine the phase space, it is shown that we can expect the magnitude of the contribution to the Gottfried sum from the soft pion is about $-0.04 \sim -0.02$. The main contribution of the soft pion to this sum comes from the small x_B region.
In the phenomenological parton model, we have not yet taken into account the soft pion contribution except when we have a general constraint such as the Adler sum rule. In such a case the soft pion contribution is effectively taken into account in the quark distributions by satisfying the sum rule. However in case of the sea quark distributions, no such sum rule constraint is imposed. We have many sum rules concerning the sea quarks[6, 18] which have a clear physical meaning, and the modified Gottfried sum rule is one example among these sum rules. It gives us information on up and down sea quarks. The mean charge sum rule for the light sea quarks stands on the same theoretical footing as the modified Gottfried sum rule. Unfortunately, all the presently available strange sea quark distribution badly breaks the mean charge sum rule,[18] and here is a soft pion contribution which has not yet been taken into account. In conclusion, combined analysis of the sum rule and the soft pion at high energy gives us an important insight into the sea quark distribution functions or the vacuum structure of the hadron.

References

[1] N.Sakai and M.Yamada, *Phys. Lett.* **37B**, 505 (1971); N.Sakai, *Nucl. Phys.* B **39**, 119 (1972).

[2] S.Koretune, *Prog. Theor. Phys.* **59**, 1989 (1978);and early references cited therein.

[3] S.Koretune, *Phys. Lett.* **115B**, 261 (1982).

[4] S.Koretune, *Phys. Rev.* D **21**, 820 (1980).

[5] S.Koretune, *Phys. Rev.* D **47**, 2690 (1993);and early references cited therein.

[6] S.Koretune, *Nucl. Phys.* B **526**, 445 (1998). S.Koretune, in *Proceedings of the 29th International Conference on High Energy Physics (ICHEP 98)*,edited by A.Astbury,D.Axen and J.Robinson (World Scientific, singapore, 1999)pp862-866.

[7] S.Koretune,Y.Masui, and M.Aoyama, *Phys. Rev.* D **18**, 3248 (1978).

[8] S.Koretune, *Prog. Theor. Phys.* **103**, 127 (2000).

[9] S.Deser, W.Gilbert and E.C.G.Sudarshan, *Phys. Rev.* **115**, 731(1959).

[10] S.Koretune , *Prog. Theor. Phys.* **72**, 821 (1984).

[11] P.Amaudruz *et al.*, *Phys. Rev. Lett.* **66**, 2712 (1991).

[12] S.Kumano, *Phys. Rept.* **303**, 183 (1998).

[13] E866Collaboration, E.A.Hawker *et al.*, *Phys. Rev. Lett.* **80**, 3715 (1998). J.C.Peng *et al.*, *Phys. Rev.* D **58**, 092004 (1998).

[14] H.Fritzsch and M.Gell-mann, in *Proceedings of the International Conference on Duality and Symmetry in Hadron Physics*, edited by E.Gotsman (Weizmann Science Physics, Jerusalem,1971)p.317.

[15] C.J.Bebek *et al.*, *Phys. Rev.* D **16**, 1986 (1977).

[16] T.Gehrmann and W.J.Stirling, *Phys. Rev.* D **53**, 6100(1996); A.D.Martin, R.G.Roberts, and W.J.Stirling, *Phys. Lett.* B **354**, 155 (1995).

[17] DELPHI collaboration, P.Abreu *et al.*, *Zeit Phys.* C **50**, 185 (1995).

[18] S.Koretune, *Prog. Theor. Phys.* **98**, 749 (1997).